Solution to Physic 9C-A MT#1 (2020; W)

1-(a) $\vec{r}_1 = 2\times10^{-2}\,m\,\hat{y}$; $\vec{r}_2 = 2\times10^{-2}\,m\cdot(-\hat{y})$

$$\vec{F}_{on\,Q} = \vec{F}_{q_1 on\,Q} + \vec{F}_{q_2 on\,Q}$$

$$= K\cdot\frac{q_1 Q}{r_1^2}\hat{r}_1 + K_2\frac{q_2 Q}{r_2^2}\hat{r}_2$$

$$= \frac{K\cdot Q}{r_1^2}(q_1 - q_2)\cdot\hat{y}$$

$$= 9.0\times10^{9}\,N\cdot m^2/C^2\cdot\frac{(-2\times10^{-9}C)(-3\times10^{-9}C)}{(2\times10^{-2}m)^2}\cdot\hat{y}$$

$$= 1.35\times10^{-4}\,N\cdot\hat{y}$$

1-(b) $\vec{r}_1 = 3\times10^{-2}\,m\,\hat{x}$; $\vec{r}_2 = 3\times10^{-2}\,m\,\hat{x} - 4\times10^{-2}\,m\cdot\hat{y}$

$$\vec{E}_B = \vec{E}_{q_1\,at\,B} + \vec{E}_{q_2\,at\,B}$$

$$= K\frac{q_1\hat{r}_1}{r_1^2} + K\frac{q_2}{r_2^2}\hat{r}_2 = K\frac{q_1}{r_1^3}\vec{r}_1 + K\frac{q_2}{r_2^3}\vec{r}_2$$

$$= K\left(\frac{q_1}{r_1^2} + \frac{q_2}{r_2^3}(3\times10^{-2}m)\right)\hat{x}$$

$$+ K\frac{q_2}{r_2^3}(-4\times10^{-2}m)\cdot\hat{y}$$

$$\therefore \; E_{B,x} = 9.0\times10^{9}\,N\cdot m^2/C^2 \cdot \frac{(-2\times10^{-9}C)}{(3\times10^{-2}m)^2}$$

$$+ 9.0\times10^{9}\,N\cdot m^2/C^2 \cdot \frac{(+10^{-9}C)\cdot(3\times10^{-2}m)}{\left((3\times10^{-2}m)^2+(-4\times10^{-2}m)^2\right)^{3/2}}$$

$$\cong -1.8\times10^{4}\,N/C$$

$$E_{B,y} = 9\times10^{9}\,N\cdot m^2/C^2 \cdot \frac{(+10^{-9}C)\cdot(-4\times10^{-2}m)}{\left((3\times10^{-2}m)^2+(-4\times10^{-2}m)^2\right)^{3/2}}$$

$$= -0.29\times10^{4}\,N/C$$

1-(C) $r_{1A} = 2\times10^{-2}m, \quad r_{1B} = 3\times10^{-2}m;$

$r_{2A} = 2\times10^{-2}m, \quad r_{2B} = \left((3\times10^{-2}m)^2+(4\times10^{-2})^2\right)^{1/2} = 5\times10^{-2}m$

$$U(Q;A) - U(Q;B) = \left(U(Q;A)-U(Q;B)\right)_{q_1} + \left(U(Q;A)-U(Q;B)\right)_{q_2}$$

$$= \left[Kq_1\left(\frac{1}{r_{1A}}-\frac{1}{r_{1B}}\right) + Kq_2\left(\frac{1}{r_{2A}}-\frac{1}{r_{2B}}\right)\right]\cdot Q$$

$6\times10^{-8}J$

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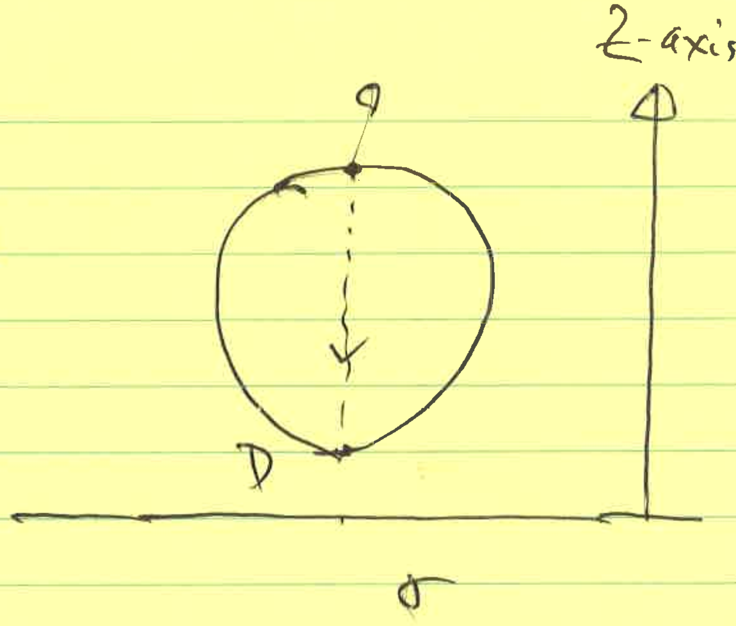
$$= 9\times10^{9}\,N\cdot m^2/C^2\left[(-2\times10^{-9}C)\left(\frac{1}{2\times10^{-2}m}-\frac{1}{3\times10^{-2}m}\right) + (10^{-9}C)\left(\frac{1}{2\times10^{-2}m}-\frac{1}{5\times10^{-2}m}\right)\right]\cdot(-2\times10^{-9}C)$$

2-(a) By definition,



$$V_C - V_D = \int_C^D \vec{E}\cdot d\vec{l}$$

$$= \int_C^D \left(\vec{E}_\sigma + \vec{E}_{shell}\right)\cdot d\vec{l}$$

$$= \int_C^D \vec{E}_\sigma \cdot d\vec{l} \qquad \left(\vec{E}_{shell} = 0\right)$$

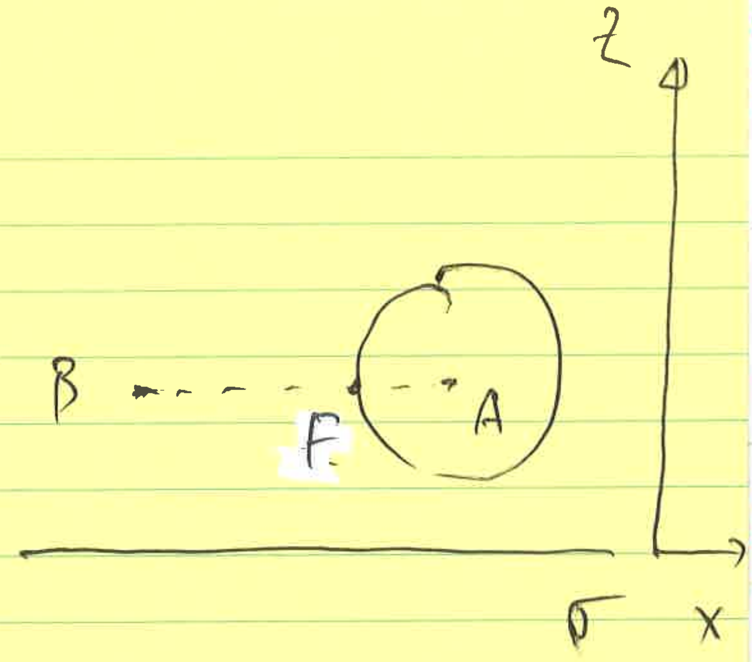
$$= (-)\cdot \int_D^C \vec{E}_\sigma \cdot d\vec{l}$$

$$= (-)\cdot \int_D^C \left(\frac{\sigma}{2\epsilon_0}\hat{z}\right)\cdot\left(\hat{z}\cdot dz\right)$$

$$= (-)\,\frac{\sigma}{2\epsilon_0}\cdot(2R)$$

$$= -\frac{\sigma\cdot R}{\epsilon_0}$$

2-(b) By definition



$$V_A - V_B = \int_A^B \vec{E}\cdot d\vec{l}$$

$$= \int_A^B \vec{E}_\sigma \cdot d\vec{l} + \int_A^B \vec{E}_{shell}\cdot d\vec{l}$$

Since B and A are at the same distance to the charge sheet, the straight path from A to B is perpendicular to the electric field by σ, then,

$$\int_A^B \vec{E}_\sigma \cdot d\vec{l} = 0$$

$$\therefore V_A - V_B = \int_A^B \vec{E}_{shell}\cdot d\vec{l} = \cancelto{0}{\int_A^F \vec{E}_{shell}\cdot d\vec{l}} + \int_F^B \vec{E}_{shell}\cdot d\vec{l}$$

$$= KQ\left(\frac{1}{R} - \frac{1}{4R}\right) = \frac{3KQ}{4R}$$

2-(c) Every piece of charge on the spherical shell experiences the same electric field from the infinite charge sheet

$$\vec{E}_\sigma = \frac{\sigma}{2\epsilon_0}\hat{z}$$

Thus the total Coulomb force on the spherical shell is simply the sum of its charge multiplied by $\vec{E}_\sigma$

$$\vec{F}_{\text{on shell}} = Q\vec{E}_\sigma = \frac{\sigma \cdot Q}{2\epsilon_0}\cdot\hat{z}$$

2-(d) There are two ways to look at it. For Coulomb forces between two point charges, they are equal in magnitude and opposite in sign. We should expect the force on the sheet exerted by the shell to be equal in magnitude but opposite in sign, thus

$$\vec{F}_{\text{on sheet}} = -\vec{F}_{\text{on shell}} = -\frac{\sigma \cdot Q}{2\epsilon_0}\cdot\hat{z}$$

Another way is to calculate these forces using the Coulomb forces between two small pieces of charges on the sheet and the shell: Δq_{sheet} and Δq_{shell}

$$\Delta \vec{F}_{\Delta q_{sheet} \text{ on } \Delta q_{shell}} = K \frac{\Delta q_{sheet} \cdot \Delta q_{shell}}{r^2_{shell-sheet}} \hat{r}_{\text{sheet-shell}}$$

$$= -\Delta \vec{F}_{\Delta q_{shell} \text{ on } \Delta q_{sheet}} = K \frac{\Delta q_{sheet} \cdot \Delta q_{shell}}{r^2_{shell-sheet}} \hat{r}_{shell-sheet}$$

$$\therefore \vec{F}_{\text{on shell}} = \sum_{\substack{\Delta q_{sheet} \\ \Delta q_{shell}}} \Delta \vec{F}_{\Delta q_{sheet} \text{ on } \Delta q_{shell}}$$

$$= -\left(\sum_{\substack{\Delta q_{shell} \\ \Delta q_{sheet}}} \Delta \vec{F}_{\Delta q_{shell} \text{ on } \Delta q_{sheet}} \right)$$

$$= -\vec{F}_{\text{on sheet}}$$

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2-(e) If the round holes are small enough, we can treat the removed pieces as two circular discs of uniform charges

At C, the total electric field is

$$\vec{E}_C = \vec{E}_\sigma(C) + \vec{E}_{shell}(C) - \vec{E}_{disc\ at\ C}(C) - \vec{E}_{disc\ at\ D}(C)$$

$$\cong \vec{E}_\sigma(C) + \vec{E}_{shell}(C) - \vec{E}_{disc\ at\ C}(C)$$

$$= \frac{\sigma}{2\epsilon_0}\hat{z} + \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}\hat{z} - \frac{1}{2}\frac{Q}{4\pi\epsilon_0 R^2}\hat{z}$$

$$= \frac{\sigma}{2\epsilon_0}\hat{z} + \frac{Q}{8\pi\epsilon_0 R^2}\hat{z}$$

$$\vec{E}_D = \vec{E}_\sigma(D) + \vec{E}_{shell}(D) - \vec{E}_{disc\ at\ C}(D) - \vec{E}_{disc\ at\ D}(D)$$

$$= \frac{\sigma}{2\epsilon_0}\hat{z} - \frac{Q}{8\pi\epsilon_0 R^2}\hat{z}$$

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3-(a) When $r<a$, $\vec{E}(\vec{r}<a)=0$ as all spherically symmetric charge distributions are at larger distance from the center.

When $b<r<a$, $\vec{E}(\vec{r}) = K\cdot\frac{(-2Q)}{r^2}\hat{r}$

when $r>d$, $\vec{E}(\vec{r}) = K\cdot\frac{(-2Q+Q)}{r^2}\hat{r}$

$$= -K\frac{Q}{r^2}\hat{r}$$

3-(b) Since the electric field inside the first conducting shell is zero, and it is given by

$$\vec{E}(a<r<b) = K\frac{Q_a}{r^2}\hat{r} = 0$$

$\therefore Q_a = 0$. This means $Q_b = -2Q$.

Since the electric field inside the second shell is also zero, and it is given by

$$\vec{E}(c<r<d) = K\frac{(-2Q)+Q_c}{r^2}\hat{r} = 0$$

$\therefore Q_c = +2Q$. This means $Q_d = -Q$

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3-(c) By definition

$$V_0 - V_\infty = \int_0^{\infty} \vec{E} \cdot d\vec{l}$$

$$= \int_0^{a} \vec{E} \cdot d\vec{l} + \int_a^{b} \vec{E} \cdot d\vec{l} + \int_b^{c} \vec{E} \cdot d\vec{l} + \int_c^{d} \vec{E} \cdot d\vec{l}$$

$$+ \int_d^{\infty} \vec{E} \cdot d\vec{l}$$

$$= \int_c^{d} \vec{E} \cdot d\vec{l} + \int_d^{+\infty} \vec{E} \cdot d\vec{l}$$

$$= k \cdot (-2Q) \left(\frac{1}{b} - \frac{1}{c} \right) + k(-Q) \cdot \left(\frac{1}{d} \right)$$

$$= kQ \left(\frac{2}{c} - \frac{2}{b} - \frac{1}{d} \right)$$

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